Output-based Mesh Adaptation for Variable-fidelity Multipoint Aerodynamic Optimization

Guodong Chen   Krzysztof J. Fidkowski

Department of Aerospace Engineering

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UNIVERSITY OF MICHIGAN

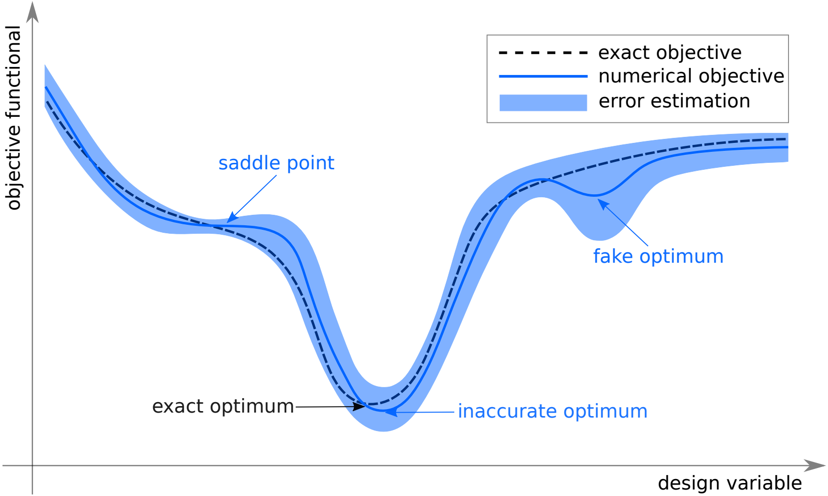
Outline

1. Motivation
2. Multipoint optimization formulation
3. Error estimation and mesh adaptation
4. Results and discussion
5. Conclusions and future work

Outline

How does discretization error affect optimization?

Aerodynamic design/optimization: numerical optimization + CFD analysis



Discretization error can have detrimental effects on optimization

Motivation in multipoint aerodynamic optimization

Single-point aerodynamic optimization:

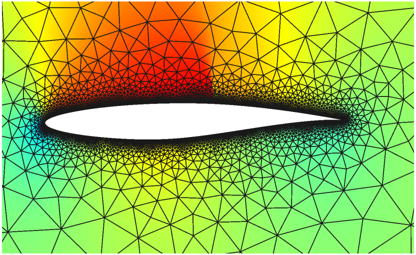
- How do we control the discretization error?
  - Fixed fine mesh ✗
  - Adapted for the initial design ✗
  - Actively-adapted mesh ✓

Multipoint aerodynamic optimization:

- Error should be controlled at each design point (flight condition)
- What mesh to use in multipoint optimization?
  - Single mesh adapted for all the flight conditions ✗
  - Multiple meshes adapted individually ✓

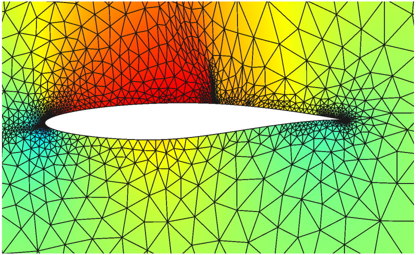
Example: fixed-lift drag minimization at $M_\infty = 0.72$

RAE 2822, *a priori* mesh



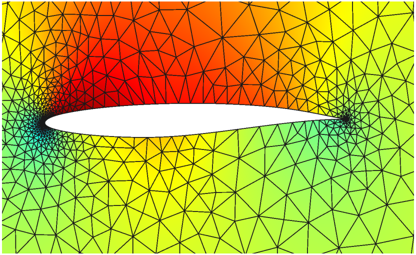
$DOF = 45254,\ \delta J = 1.13 \times 10^{-5}$

RAE 2822, adapted mesh



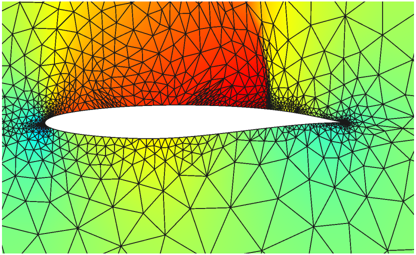
$DOF = 26502,\ \delta J = 9.78 \times 10^{-7}$

optimized design, adapted mesh



Shock-free, $DOF = 17290,\ \delta J = 8.05 \times 10^{-7}$

off-design point, adapted mesh



$M_\infty = 0.76$, $DOF = 20280,\ \delta J = 3.02 \times 10^{-7}$

Outline

Multipoint aerodynamic optimization problem

- Determine the design variables $\mathbf{x}$ that minimize the composite objective function J_m^{adapt}:

$$\begin{aligned} \min_{\mathbf{x}} \quad & J_m^{\text{adapt}} = \sum_{i=1}^{N_m} \omega_i J_i^{\text{adapt}}(\mathbf{U}_i, \mathbf{x}) \\ \text{s.t.} \quad & \mathbf{R}_i(\mathbf{U}_i, \mathbf{x}) = \mathbf{0} \\ & \mathbf{R}_i^{\text{trim}}(\mathbf{U}_i, \mathbf{x}) = \mathbf{J}_i^{\text{trim}} - \bar{\mathbf{J}}_i^{\text{trim}} = \mathbf{0} \end{aligned}$$

- N_m: number of design points
- J_i^{adapt}: adapt (objective) output, direct target for adaptation, e.g., c_d
- $\mathbf{J}_i^{\text{trim}}$: trim outputs, indirectly affect objective, e.g., c_ℓ
- $\bar{\mathbf{J}}_i^{\text{trim}}$: constant target values for trim outputs
- $\mathbf{R}_i(\mathbf{U}_i, \mathbf{x})$: flow governing equations; $\mathbf{U}_i$: flow states vector

Adjoint and design equations

Lagrangian function:

$$\mathcal{L} = \sum_{i=1}^{N_m} \omega_i J_i^{\text{adapt}}(\mathbf{U}_i, \mathbf{x}) + \sum_{i=1}^{N_m} \boldsymbol{\lambda}_i^T \mathbf{R}_i(\mathbf{U}_i, \mathbf{x}) + \sum_{i=1}^{N_m} \boldsymbol{\mu}_i^T \mathbf{R}_i^{\text{trim}}(\mathbf{U}_i, \mathbf{x})$$

First order optimality (KKT) condition:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}} = \sum_{i=1}^{N_m} \omega_i \frac{\partial J_i^{\text{adapt}}}{\partial \mathbf{x}} + \sum_{i=1}^{N_m} \boldsymbol{\lambda}_i^T \frac{\partial \mathbf{R}_i}{\partial \mathbf{x}} + \sum_{i=1}^{N_m} \boldsymbol{\mu}_i^T \frac{\partial \mathbf{R}_i^{\text{trim}}}{\partial \mathbf{x}} = \mathbf{0} \qquad \text{design equations}$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{U}_i} = \omega_i \frac{\partial J_i^{\text{adapt}}}{\partial \mathbf{U}_i} + \boldsymbol{\lambda}_i^T \frac{\partial \mathbf{R}_i}{\partial \mathbf{U}_i} + \boldsymbol{\mu}_i^T \frac{\partial \mathbf{R}_i^{\text{trim}}}{\partial \mathbf{U}_i} = \mathbf{0} \qquad \text{adjoint equations}$$

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{\lambda}_i} = \mathbf{R}_i(\mathbf{U}_i, \mathbf{x}) = \mathbf{0} \qquad \text{flow equations}$$

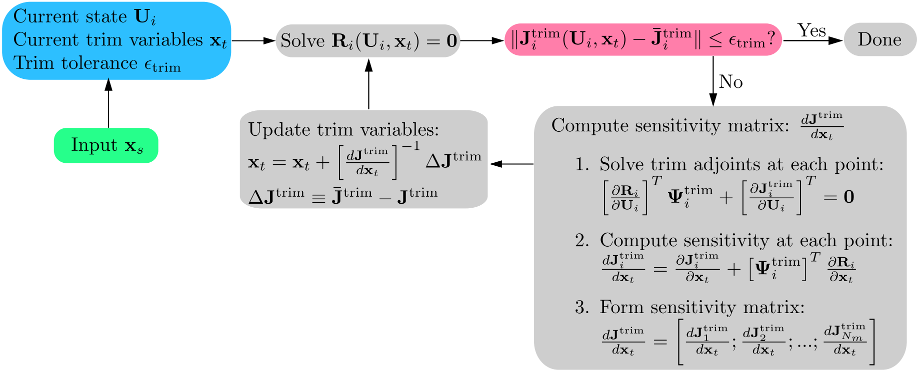
$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{\mu}_i} = \mathbf{R}_i^{\text{trim}}(\mathbf{U}_i, \mathbf{x}) = \mathbf{0} \qquad \text{trim equations}$$

Reduced optimization problem

Decompose the design space $\mathbf{x} = [\mathbf{x}_t, \mathbf{x}_s]^T$, given active design $\mathbf{x}_s$, trimming with trim variables $\mathbf{x}_t \Rightarrow \mathbf{R}_i^{\text{trim}}(\mathbf{U}_i, \mathbf{x}_t) = \mathbf{0}, \dim(\mathbf{x}_t) = \sum \dim(\mathbf{R}_i^{\text{trim}}) = \dim(\mathbf{R}^{\text{trim}})$

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{\lambda}_i} = \mathbf{R}_i(\mathbf{U}_i, \mathbf{x}_t, \mathbf{x}_s) = \mathbf{0} \qquad \text{~~flow equations~~}$$

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{\mu}_i} = \mathbf{R}_i^{\text{trim}}(\mathbf{U}_i, \mathbf{x}_t, \mathbf{x}_s) = \mathbf{0} \qquad \text{~~trim equations~~}$$



Reduced optimization problem

Reduced optimality condition:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}_s} = \sum_{i=1}^{N_m} \omega_i \frac{\partial J_i^{\text{adapt}}}{\partial \mathbf{x}_s} + \sum_{i=1}^{N_m} \boldsymbol{\lambda}_i^T \frac{\partial \mathbf{R}_i}{\partial \mathbf{x}_s} + \sum_{i=1}^{N_m} \boldsymbol{\mu}_i^T \frac{\partial \mathbf{R}_i^{\text{trim}}}{\partial \mathbf{x}_s} = \mathbf{0} \qquad \text{active variables}$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}_t} = \sum_{i=1}^{N_m} \omega_i \frac{\partial J_i^{\text{adapt}}}{\partial \mathbf{x}_t} + \sum_{i=1}^{N_m} \boldsymbol{\lambda}_i^T \frac{\partial \mathbf{R}_i}{\partial \mathbf{x}_t} + \sum_{i=1}^{N_m} \boldsymbol{\mu}_i^T \frac{\partial \mathbf{R}_i^{\text{trim}}}{\partial \mathbf{x}_t} = \mathbf{0} \qquad \text{~~coupled adjoints~~}$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{U}_i} = \omega_i \frac{\partial J_i^{\text{adapt}}}{\partial \mathbf{U}_i} + \boldsymbol{\lambda}_i^T \frac{\partial \mathbf{R}_i}{\partial \mathbf{U}_i} + \boldsymbol{\mu}_i^T \frac{\partial \mathbf{R}_i^{\text{trim}}}{\partial \mathbf{U}_i} = \mathbf{0} \qquad \text{~~coupled adjoints~~}$$

Solve for coupled adjoints:

$$\boldsymbol{\lambda}_i^T = -\left(\omega_i \frac{\partial J_i^{\text{adapt}}}{\partial \mathbf{U}_i} + \boldsymbol{\mu}_i^T \frac{\partial \mathbf{R}_i^{\text{trim}}}{\partial \mathbf{U}_i}\right) \frac{\partial \mathbf{R}_i}{\partial \mathbf{U}_i}^{-1} = (\omega_i \boldsymbol{\Psi}_i^{\text{adapt}} + \boldsymbol{\Psi}_i^{\text{trim}} \boldsymbol{\mu}_i)^T$$

$$\boldsymbol{\mu}^T = [\boldsymbol{\mu}_1^T, \ldots, \boldsymbol{\mu}_{N_m}^T] = -\left(\sum_{i=1}^{N_m} \omega_i \frac{dJ_i^{\text{adapt}}}{d\mathbf{x}_t}\right) \left(\frac{d\mathbf{J}^{\text{trim}}}{d\mathbf{x}_t}\right)^{-1}$$

Reduced optimization problem

Coupled adjoints:

$$\boldsymbol{\lambda}_i^T = -\left(\omega_i \frac{\partial J_i^{\text{adapt}}}{\partial \mathbf{U}_i} + \boldsymbol{\mu}_i^T \frac{\partial \mathbf{R}_i^{\text{trim}}}{\partial \mathbf{U}_i}\right) \frac{\partial \mathbf{R}_i}{\partial \mathbf{U}_i}^{-1} = (\omega_i \boldsymbol{\Psi}_i^{\text{adapt}} + \boldsymbol{\Psi}_i^{\text{trim}} \boldsymbol{\mu}_i)^T$$

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Reduced optimality condition:

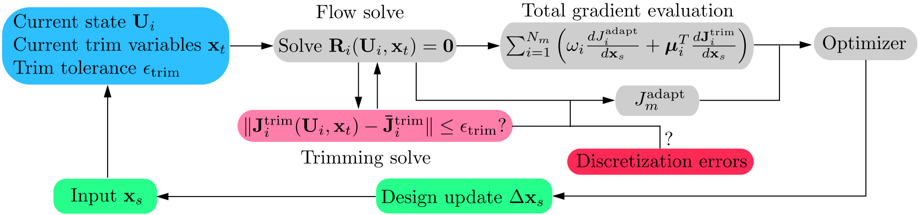
$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \mathbf{x}_s} &= \sum_{i=1}^{N_m} \omega_i \frac{\partial J_i^{\text{adapt}}}{\partial \mathbf{x}_s} + \sum_{i=1}^{N_m} \boldsymbol{\lambda}_i^T \frac{\partial \mathbf{R}_i}{\partial \mathbf{x}_s} + \sum_{i=1}^{N_m} \boldsymbol{\mu}_i^T \frac{\partial \mathbf{R}_i^{\text{trim}}}{\partial \mathbf{x}_s} \\
&= \sum_{i=1}^{N_m} \omega_i \frac{\partial J_i^{\text{adapt}}}{\partial \mathbf{x}_s} + \sum_{i=1}^{N_m} [\omega_i (\boldsymbol{\Psi}_i^{\text{adapt}})^T + \boldsymbol{\mu}_i^T (\boldsymbol{\Psi}_i^{\text{trim}})^T] \frac{\partial \mathbf{R}_i}{\partial \mathbf{x}_s} + \sum_{i=1}^{N_m} \boldsymbol{\mu}_i^T \frac{\partial \mathbf{R}_i^{\text{trim}}}{\partial \mathbf{x}_s} \\
&= \sum_{i=1}^{N_m} \omega_i \left[\frac{\partial J_i^{\text{adapt}}}{\partial \mathbf{x}_s} + (\boldsymbol{\Psi}_i^{\text{adapt}})^T \frac{\partial \mathbf{R}_i}{\partial \mathbf{x}_s}\right] + \sum_{i=1}^{N_m} \boldsymbol{\mu}_i^T \left[\frac{\partial \mathbf{J}_i^{\text{trim}}}{\partial \mathbf{x}_s} + (\boldsymbol{\Psi}_i^{\text{trim}})^T \frac{\partial \mathbf{R}_i}{\partial \mathbf{x}_s}\right] \\
&= \sum_{i=1}^{N_m} \omega_i \frac{dJ_i^{\text{adapt}}}{d\mathbf{x}_s} + \sum_{i=1}^{N_m} \boldsymbol{\mu}_i^T \frac{d\mathbf{J}_i^{\text{trim}}}{d\mathbf{x}_s} = \mathbf{0} \qquad \text{active variables}
\end{aligned}$$

Reduced optimization problem

Coupled adjoints:

$$\boldsymbol{\lambda}_i^T = -\left(\omega_i \frac{\partial J_i^{\text{adapt}}}{\partial \mathbf{U}_i} + \boldsymbol{\mu}_i^T \frac{\partial \mathbf{R}_i^{\text{trim}}}{\partial \mathbf{U}_i}\right) \frac{\partial \mathbf{R}_i}{\partial \mathbf{U}_i}^{-1} = (\omega_i \boldsymbol{\Psi}_i^{\text{adapt}} + \boldsymbol{\Psi}_i^{\text{trim}} \boldsymbol{\mu}_i)^T$$

$$\boldsymbol{\mu}^T = [\boldsymbol{\mu}_1^T, \ldots, \boldsymbol{\mu}_{N_m}^T] = -\left(\sum_{i=1}^{N_m} \omega_i \frac{dJ_i^{\text{adapt}}}{d\mathbf{x}_t}\right) \left(\frac{d\mathbf{J}^{\text{trim}}}{d\mathbf{x}_t}\right)^{-1}$$



Optimizer drives the gradient to zero (to optimization tolerance ϵ_{opt}):

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}_s} = \underbrace{\sum_{i=1}^{N_m} \omega_i \frac{dJ_i^{\text{adapt}}}{d\mathbf{x}_s} + \sum_{i=1}^{N_m} \boldsymbol{\mu}_i^T \frac{d\mathbf{J}_i^{\text{trim}}}{d\mathbf{x}_s}}_{\text{plug in both } \boldsymbol{\lambda}_i \text{ and } \boldsymbol{\mu}_i} = \mathbf{0}$$

Outline

Error estimation

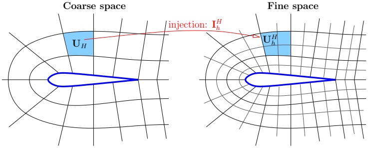
Consider a given design $\mathbf{x}_s$

$$\text{coarse space: } \mathbf{x}_{t,0} \rightarrow \text{trimming} \rightarrow \underbrace{\mathbf{x}_{t,H}, \mathbf{U}_H}_{\text{inexact trim satisfaction}} \rightarrow J_H(\mathbf{U}_H, \mathbf{x}_{t,H})$$

$$\text{fine space: } \mathbf{x}_{t,0} \rightarrow \text{trimming} \rightarrow \underbrace{\mathbf{x}_{t,h}, \mathbf{U}_h}_{\text{exact trim satisfaction}} \rightarrow J_h(\mathbf{U}_h, \mathbf{x}_{t,h})$$

Error estimate for the adapt outputs

$$\delta J_m^{\text{adapt}} = \sum_{i=1}^{N_m} \omega_i \left[J_{H,i}^{\text{adapt}}(\mathbf{U}_{H,i}, \mathbf{x}_{t,H}) - J_{h,i}^{\text{adapt}}(\mathbf{U}_{h,i}, \mathbf{x}_{t,h}) \right]$$
$$= \sum_{i=1}^{N_m} \omega_i \left[J_{h,i}^{\text{adapt}}(\mathbf{U}_{h,i}^H, \mathbf{x}_{t,H}) - J_{h,i}^{\text{adapt}}(\mathbf{U}_{h,i}, \mathbf{x}_{t,h}) \right]$$



Fine space flow residual, $\mathbf{R}_{h,i}(\mathbf{U}_{h,i}^H, \mathbf{x}_{t,H}) \neq \mathbf{0}$

Fine space trim residual

$$\mathbf{R}_{h,i}^{\text{trim}}(\mathbf{U}_{h,i}^H, \mathbf{x}_{t,H}) = \mathbf{J}_{h,i}^{\text{trim}}(\mathbf{U}_{h,i}^H, \mathbf{x}_{t,H}) - \bar{\mathbf{J}}^{\text{trim}} = \mathbf{J}_{H,i}^{\text{trim}}(\mathbf{U}_{H,i}, \mathbf{x}_{t,H}) - \bar{\mathbf{J}}^{\text{trim}} = \mathbf{0}$$

Error estimation

Recall the optimality condition:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}_t} = \sum_{i=1}^{N_m} \omega_i \frac{\partial J_i^{\text{adapt}}}{\partial \mathbf{x}_t} + \sum_{i=1}^{N_m} \boldsymbol{\lambda}_i^T \frac{\partial \mathbf{R}_i}{\partial \mathbf{x}_t} + \sum_{i=1}^{N_m} \boldsymbol{\mu}_i^T \frac{\partial \mathbf{R}_i^{\text{trim}}}{\partial \mathbf{x}_t} = \mathbf{0}$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{U}_i} = \omega_i \frac{\partial J_i^{\text{adapt}}}{\partial \mathbf{U}_i} + \boldsymbol{\lambda}_i^T \frac{\partial \mathbf{R}_i}{\partial \mathbf{U}_i} + \boldsymbol{\mu}_i^T \frac{\partial \mathbf{R}_i^{\text{trim}}}{\partial \mathbf{U}_i} = \mathbf{0}$$

Error estimation

Recall the optimality condition:

$$\sum_{i=1}^{N_m} \omega_i \frac{\partial J_i^{\text{adapt}}}{\partial \mathbf{x}_t} = -\sum_{i=1}^{N_m} \boldsymbol{\lambda}_i^T \frac{\partial \mathbf{R}_i}{\partial \mathbf{x}_t} - \sum_{i=1}^{N_m} \boldsymbol{\mu}_i^T \frac{\partial \mathbf{R}_i^{\text{trim}}}{\partial \mathbf{x}_t}$$

$$\omega_i \frac{\partial J_i^{\text{adapt}}}{\partial \mathbf{U}_i} = -\boldsymbol{\lambda}_i^T \frac{\partial \mathbf{R}_i}{\partial \mathbf{U}_i} - \boldsymbol{\mu}_i^T \frac{\partial \mathbf{R}_i^{\text{trim}}}{\partial \mathbf{U}_i}$$

Error estimation

Recall the optimality condition:

$$\sum_{i=1}^{N_m} \omega_i \frac{\partial J_i^{\text{adapt}}}{\partial \mathbf{x}_t} = -\sum_{i=1}^{N_m} \boldsymbol{\lambda}_i^T \frac{\partial \mathbf{R}_i}{\partial \mathbf{x}_t} - \sum_{i=1}^{N_m} \boldsymbol{\mu}_i^T \frac{\partial \mathbf{R}_i^{\text{trim}}}{\partial \mathbf{x}_t}$$

$$\omega_i \frac{\partial J_i^{\text{adapt}}}{\partial \mathbf{U}_i} = -\boldsymbol{\lambda}_i^T \frac{\partial \mathbf{R}_i}{\partial \mathbf{U}_i} - \boldsymbol{\mu}_i^T \frac{\partial \mathbf{R}_i^{\text{trim}}}{\partial \mathbf{U}_i}$$

Error estimation using coupled adjoints

$$\begin{aligned}
\delta J_m^{\text{adapt}} &= \sum_{i=1}^{N_m} \omega_i \left[J_{h,i}^{\text{adapt}}(\mathbf{U}_{h,i}^H, \mathbf{x}_{t,H}) - J_{h,i}^{\text{adapt}}(\mathbf{U}_{h,i}, \mathbf{x}_{t,h}) \right] \\
&= \sum_{i=1}^{N_m} \omega_i \left[\frac{\partial J_{h,i}^{\text{adapt}}}{\partial \mathbf{U}_i} \delta \mathbf{U}_i + \frac{\partial J_{h,i}^{\text{adapt}}}{\partial \mathbf{x}_t} \delta \mathbf{x}_t \right] \\
&= -\sum_{i=1}^{N_m} (\boldsymbol{\lambda}_{h,i}^T \delta \mathbf{R}_{h,i} + \boldsymbol{\mu}_{h,i}^T \delta \mathbf{R}_{h,i}^{\text{trim}})
\end{aligned}$$

Error estimation

Recall the optimality condition:

$$\sum_{i=1}^{N_m} \omega_i \frac{\partial J_i^{\text{adapt}}}{\partial \mathbf{x}_t} = -\sum_{i=1}^{N_m} \boldsymbol{\lambda}_i^T \frac{\partial \mathbf{R}_i}{\partial \mathbf{x}_t} - \sum_{i=1}^{N_m} \boldsymbol{\mu}_i^T \frac{\partial \mathbf{R}_i^{\text{trim}}}{\partial \mathbf{x}_t}$$

$$\omega_i \frac{\partial J_i^{\text{adapt}}}{\partial \mathbf{U}_i} = -\boldsymbol{\lambda}_i^T \frac{\partial \mathbf{R}_i}{\partial \mathbf{U}_i} - \boldsymbol{\mu}_i^T \frac{\partial \mathbf{R}_i^{\text{trim}}}{\partial \mathbf{U}_i}$$

Error estimation using coupled adjoints

$$\begin{aligned}
\delta J_m^{\text{adapt}} &= \sum_{i=1}^{N_m} \omega_i \left[J_{h,i}^{\text{adapt}}(\mathbf{U}_{h,i}^H, \mathbf{x}_{t,H}) - J_{h,i}^{\text{adapt}}(\mathbf{U}_{h,i}, \mathbf{x}_{t,h}) \right] \\
&= \sum_{i=1}^{N_m} \omega_i \left[\frac{\partial J_{h,i}^{\text{adapt}}}{\partial \mathbf{U}_i} \delta \mathbf{U}_i + \frac{\partial J_{h,i}^{\text{adapt}}}{\partial \mathbf{x}_t} \delta \mathbf{x}_t \right] \\
&= -\sum_{i=1}^{N_m} \left(\boldsymbol{\lambda}_{h,i}^T \delta \mathbf{R}_{h,i} + \cancelto{0}{\boldsymbol{\mu}_{h,i}^T \delta \mathbf{R}_{h,i}^{\text{trim}}} \right) \\
&= -\sum_{i=1}^{N_m} \left[\boldsymbol{\lambda}_{h,i}^T \mathbf{R}_{h,i}(\mathbf{U}_{h,i}^H, \mathbf{x}_{t,H}) \right]
\end{aligned}$$

Mesh adaptation (error-based)

$$\delta J_m^{\text{adapt}} = -\sum_{i=1}^{N_m} \left[\boldsymbol{\lambda}_{h,i}^T \mathbf{R}_{h,i}(\mathbf{U}_{h,i}^H, \mathbf{x}_{t,H}) \right] = -\sum_{i=1}^{N_m} (\omega_i \boldsymbol{\Psi}_{h,i}^{\text{adapt}} + \boldsymbol{\Psi}_{h,i}^{\text{trim}} \boldsymbol{\mu}_{h,i})^T \mathbf{R}_{h,i}(\mathbf{U}_{h,i}^H, \mathbf{x}_{t,H})$$

$$= \sum_{i=1}^{N_m} (\omega_i \delta J_i^{\text{adapt}} + \boldsymbol{\mu}_{h,i}^T \delta \mathbf{J}_i^{\text{trim}}) = \sum_{i=1}^{N_m} \delta J_{m,i}^{\text{adapt}} \Rightarrow \text{Localize to each design point}$$

$$\leq \sum_{i=1}^{N_m} \sum_{e=1}^{N_{e,i}} \omega_i |(\boldsymbol{\Psi}_{i,e}^{\text{adapt}})^T \mathbf{R}_{i,e}| + |\boldsymbol{\mu}_i^T||(\boldsymbol{\Psi}_{i,e}^{\text{trim}})^T \mathbf{R}_{i,e}|, \quad \text{omit subscript } h$$

$$= \sum_{i=1}^{N_m} \sum_{e=1}^{N_{e,i}} \eta_{i,e} \Rightarrow \text{Localize to each mesh element}$$

Mesh adaptation can be either error-based or cost based:

Given error tolerance $\mathcal{E}$, first equally distribute the maximum allowable error,

$$\max(\delta J_{m,i}^{\text{adapt}}) \leq \mathcal{E}_i = \mathcal{E}/N_m$$

At each design point, refine the mesh until the error estimate drops below the tolerance. We use Hessian-based goal-oriented mesh adaptation in this work.

Mesh adaptation (cost-based)

Given a total cost C, we can also optimize the meshes to reduce error. How should we distribute the cost? Assume a priori error-cost model,

$$|\delta J_{m,i}^{\text{adapt}}| \propto C_i^{-(2p+1)/d} \Rightarrow \delta J_{m,i}^{\text{adapt}} = \delta J_{m,i}^{\text{adapt},0} \left(\frac{C_i}{C_i^0} \right)^{-(2p+1)/d},$$

Error equidistribution requires,

$$f_i = \frac{C_i}{C_1} = \frac{C_i^0}{C_1^0} \left[\frac{\delta J_{m,i}^{\text{adapt},0}}{\delta J_{m,1}^{\text{adapt},0}} \right]^{\frac{1}{(2p+1)/d}}, \quad i = 1, 2, ..., N_m$$

Redistribute the cost with desired cost ratios,

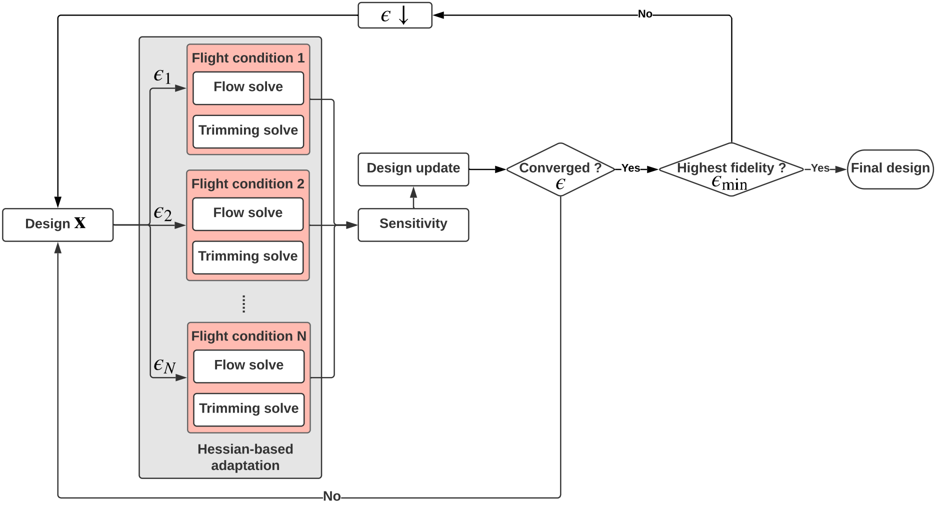
$$C_i = \frac{f_i}{\sum_{j=1}^{N_m} f_j} C$$

At each point, optimize the mesh: perform mesh optimization via error sampling and synthesis (MOESS)

$$\min_{\mathcal{T}_i} \quad \sum_e \eta_{i,e} \qquad \text{s.t.} \;\; C_i = \text{const}$$

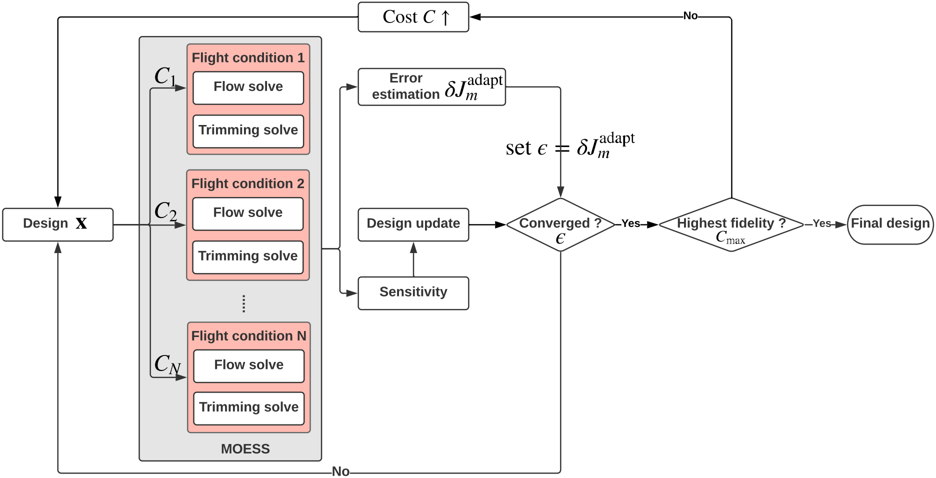
Adaptive multi-fidelity optimization

Error-based multi-fidelity optimization:
max allowable objective error $\downarrow \Rightarrow$ mesh refinement $\Rightarrow$ fidelity $\uparrow$



Adaptive multi-fidelity optimization

Cost-based multi-fidelity optimization:
given cost $\uparrow \Rightarrow$ mesh optimization $\Rightarrow$ objective error $\downarrow \Rightarrow$ fidelity $\uparrow$



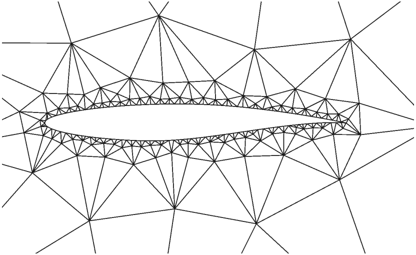
Outline

Testing case

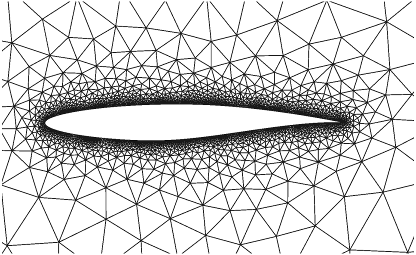
- Average drag minimization $J_{m,i}^{\text{adapt}} = c_{d,i}$, $\omega_i = 1/N_m$
- Lift trimming constraints: $c_{\ell,i} = \bar{c}_{\ell,i}$
- Airfoil parameterization: Hicks-Henne basis function
- Design parameters: airfoil shape (shared) + angle of attack (separate)
- Discretization: discontinuous Galerkin (DG) $p = 2$, cubic curved boundary elements
- Mesh movement: inverse distance weight interpolation
- Optimization algorithm: sequential least squares programming (SLSQP)

	Function/Variable	Description	Quantity
Minimize	$\sum_{i=1}^{N_m} \omega_i c_{d,i}$	weighted drag sum	1
With respect to	$\mathbf{x}_s$	shape parameters	n_s
	$\mathbf{x}_t$	angles of attack	N_m
Subject to	$c_{\ell,i} - \bar{c}_{\ell,i} = 0$	lift constraints	N_m
	$A - A_{\min} \geq 0$	volume constraint	1

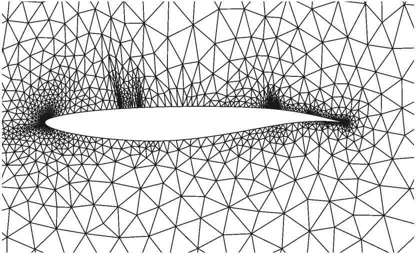
Inviscid RAE 2822, $M_{1,\infty} = 0.72$, $M_{2,\infty} = 0.76$, $J^{\text{adapt}} = c_d$, $J^{\text{trim}} = c_l$, $\bar{J}_1^{\text{trim}} = \bar{J}_2^{\text{trim}} = 0.75$, $A \geq 0.95A_0$



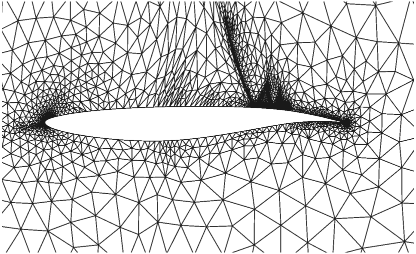
initial mesh for multifidelity optimization



fixed mesh, coarse

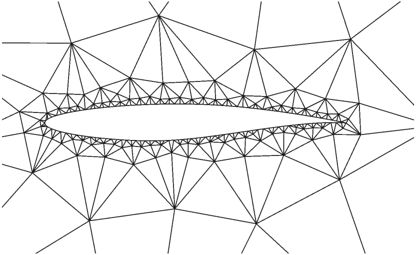


final mesh at $M = 0.72$, Hessian-based

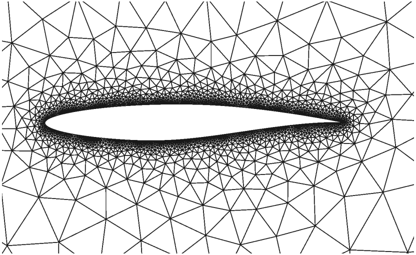


final mesh at $M = 0.76$, Hessian-based

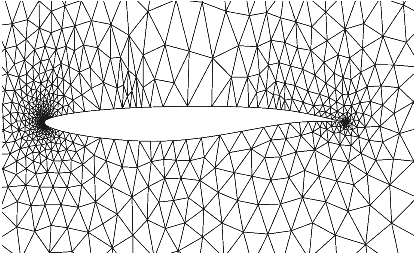
Inviscid RAE 2822, $M_{1,\infty} = 0.72$, $M_{2,\infty} = 0.76$, $J^{\text{adapt}} = c_d$, $J^{\text{trim}} = c_l$, $\bar{J}_1^{\text{trim}} = \bar{J}_2^{\text{trim}} = 0.75$, $A \geq 0.95A_0$



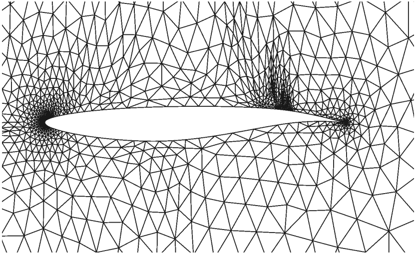
initial mesh for multifidelity optimization



fixed mesh, coarse

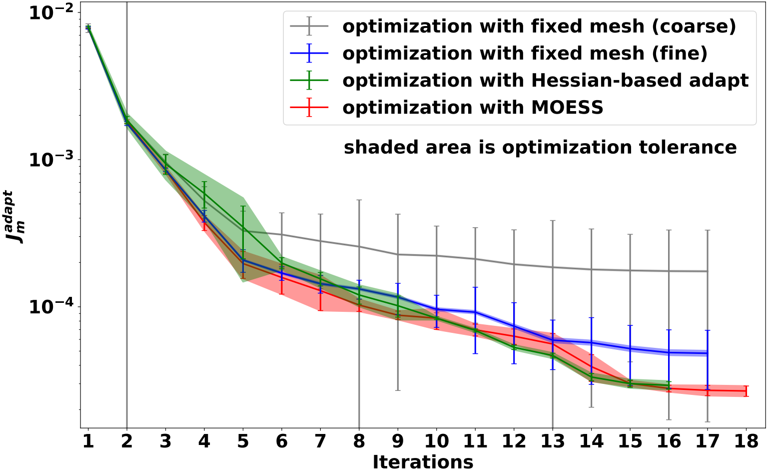


final mesh at $M = 0.72$, MOESS



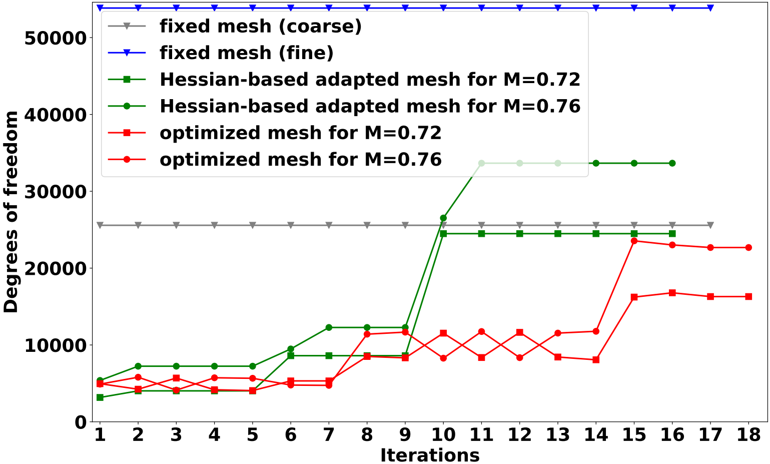
final mesh at $M = 0.76$, MOESS

Inviscid two-point transonic airfoil optimization



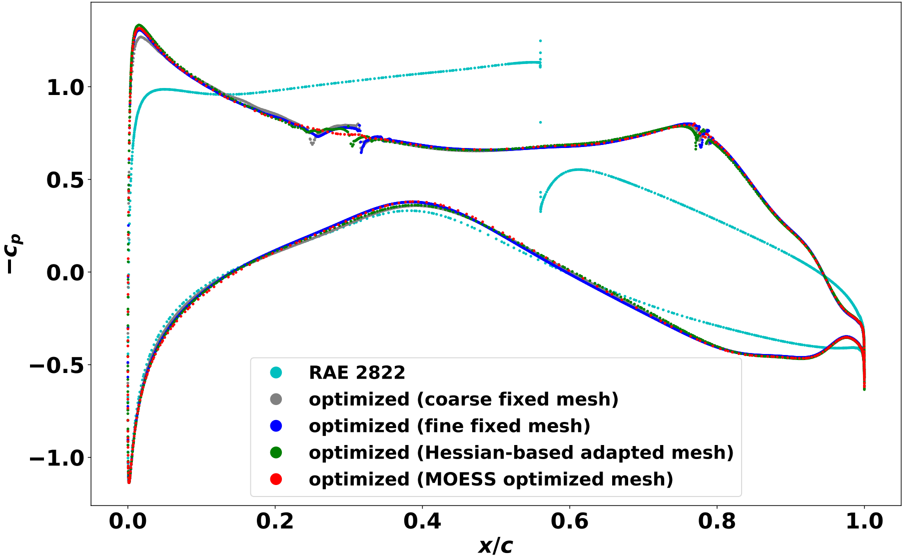
	J_m^{adapt}	J_m^{adapt} ("true")
Fixed (coarse)	$1.742 \times 10^{-4} \pm 1.577 \times 10^{-4}$	3.973×10^{-5}
Fixed (fine)	$4.833 \times 10^{-5} \pm 2.090 \times 10^{-5}$	2.778×10^{-5}
Hessian-based	$2.927 \times 10^{-5} \pm 1.727 \times 10^{-6}$	2.765×10^{-5}
MOESS	$2.679 \times 10^{-5} \pm 1.458 \times 10^{-6}$	2.533×10^{-5}

Inviscid two-point transonic airfoil optimization



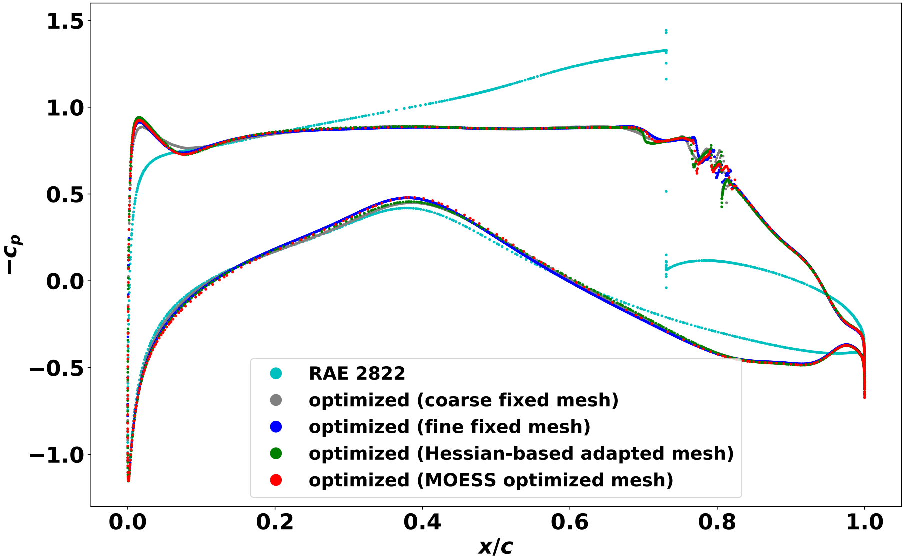
	Final total dof	Wall time (CPU*hours)
Fixed (coarse)	51144	164.67
Fixed (fine)	107688	572.67
Hessian-based	58140	88.26
MOESS	38976	31.60

Inviscid two-point transonic airfoil optimization



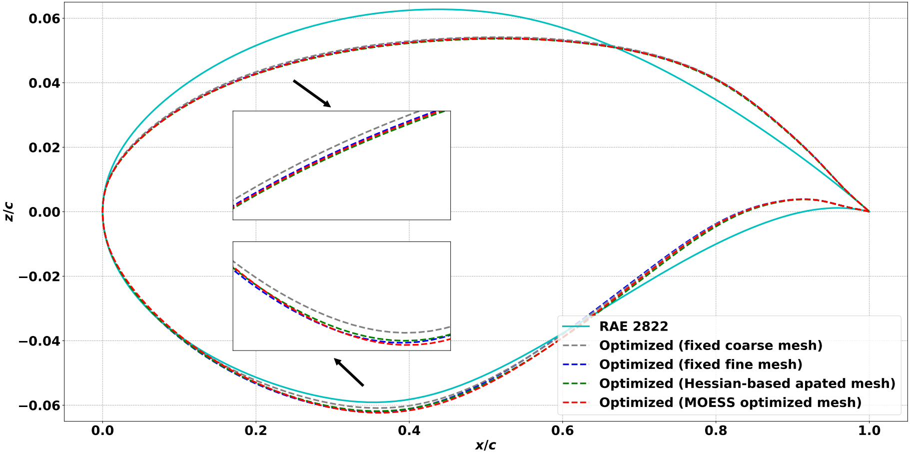
pressure distribution at $M = 0.72$

Inviscid two-point transonic airfoil optimization



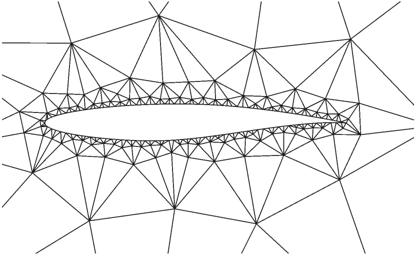
pressure distribution at $M = 0.76$

Inviscid two-point transonic airfoil optimization

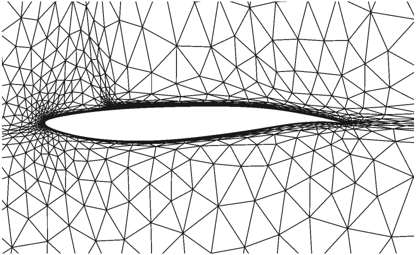


optimized shapes

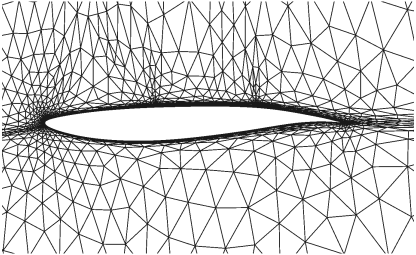
Turbulent RAE 2822, $M_{1,\infty} = 0.72$, $M_{2,\infty} = 0.74$, $M_{3,\infty} = 0.76$
$J^{\mathrm{adapt}} = c_d$, $J^{\mathrm{trim}} = c_l$, $\bar{J}_1^{\mathrm{trim}} = \bar{J}_2^{\mathrm{trim}} = \bar{J}_2^{\mathrm{trim}} = 0.75$, $A \geq 0.95 A_0$



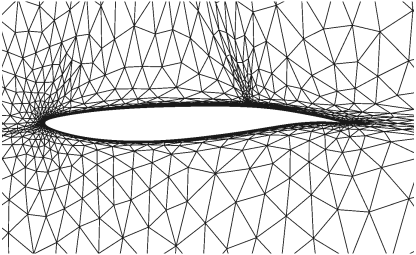
initial mesh



final mesh at $M = 0.72$

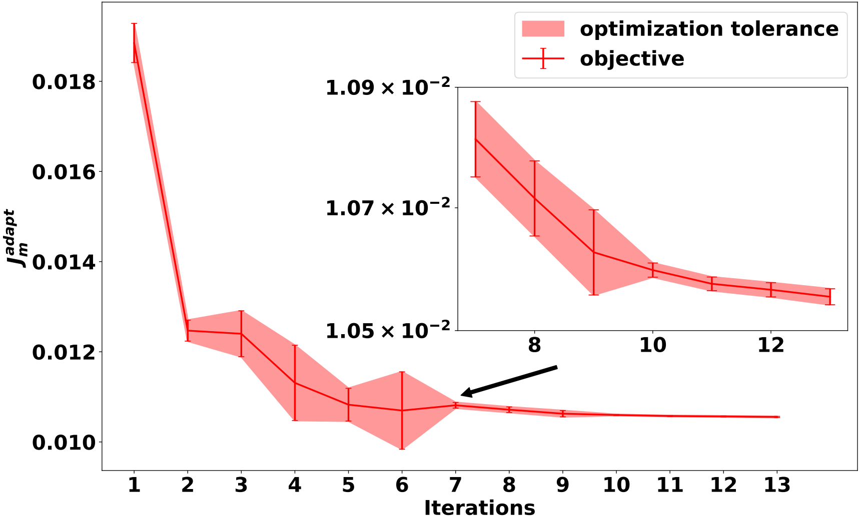


final mesh at $M = 0.74$



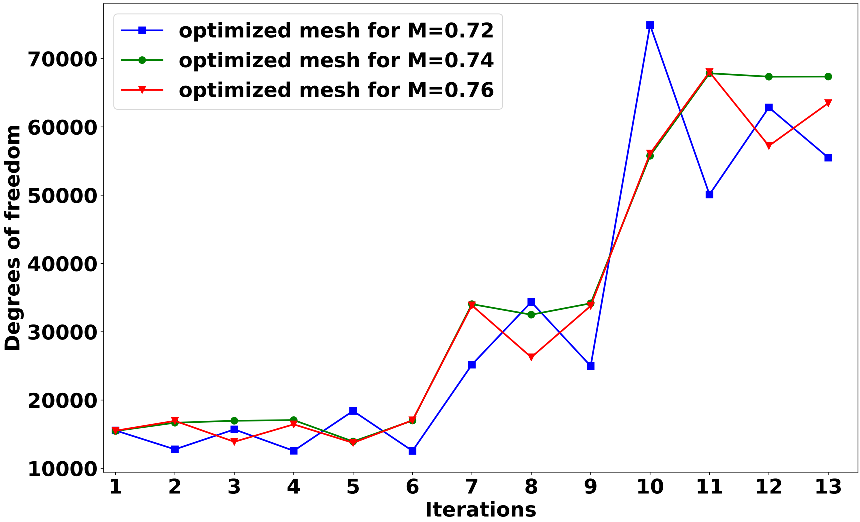
final mesh at $M = 0.76$

Turbulent three-point transonic airfoil optimization



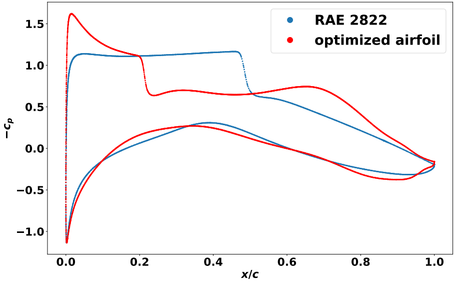
objective convergence history

Turbulent three-point transonic airfoil optimization

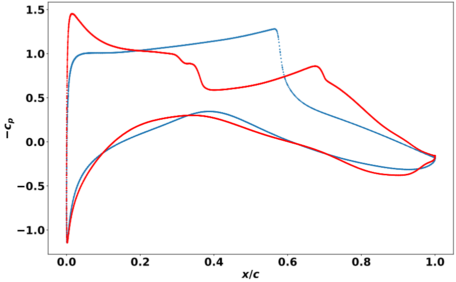


mesh size evolution

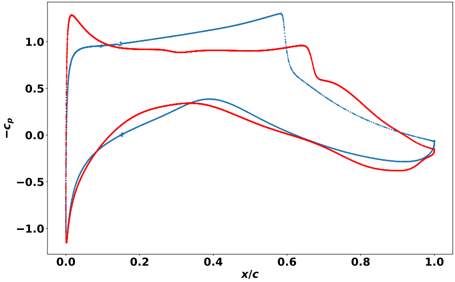
Turbulent three-point transonic airfoil optimization



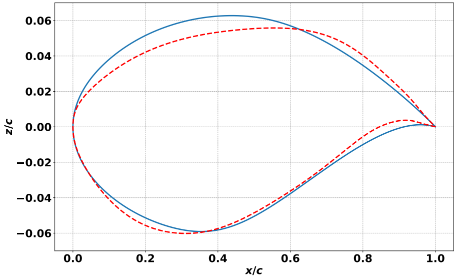
pressure distribution at $M = 0.72$



pressure distribution at $M = 0.74$

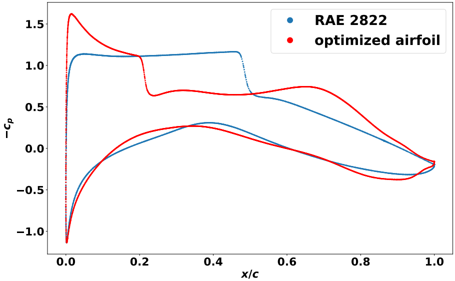


pressure distribution at $M = 0.76$

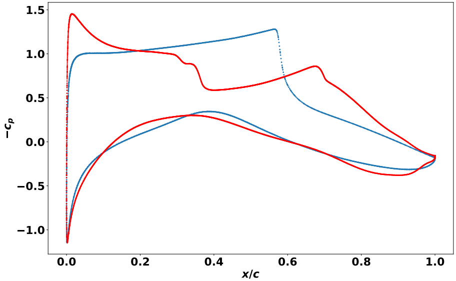


initial and final designs

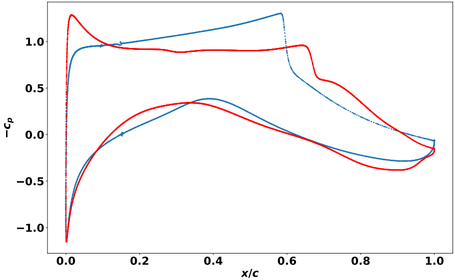
Turbulent three-point transonic airfoil optimization



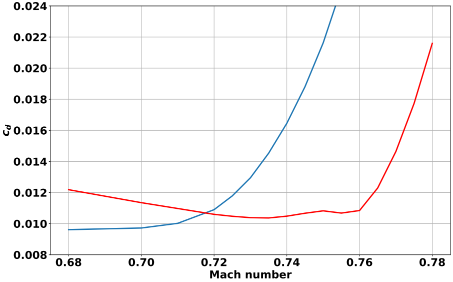
pressure distribution at $M = 0.72$



pressure distribution at $M = 0.74$



pressure distribution at $M = 0.76$



drag coefficients c_d at $\overline{J}^{\mathrm{trim}} = \bar{c}_\ell = 0.75$

Outline

Conclusions and future work

Conclusions:

- Traditional multipoint aerodynamic optimization
  - *A priori* mesh $\Rightarrow$ long set up time
  - Generated based on initial design $\Rightarrow$ insufficient/redundant
  - Numerical error not controlled $\Rightarrow$ incorrect/inaccurate design
- Proposed method: variable-fidelity optimization with error control
  - Start with fairly coarse mesh $\Rightarrow$ fast/easy set up
  - Prevents over-refining and over-optimizing $\Rightarrow$ efficient
  - Actively controls the numerical error $\Rightarrow$ accurate

Future Work:

- More appropriate error/cost distribution among different flight conditions, e.g., equidistribute error-cost ratios
- Combined with h-p refinement
- Adaptive shape parameterization together with adaptive meshes

Acknowledgments

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— Thank you —